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Mouth Organ: A Musical Cantilever

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Abstract. At first sight, a mouth organ appears not to be connected in any way with a mechanical device like a cantilever. But by treating each plate associated with a particular note in a mouth organ as a cantilever and applying the formula of the cantilever in calculating the frequency we have shown that, in fact, the ratio of any two frequencies has remarkable agreement with the experimental value. Again, the frequency of a note calculated on the basis of cantilever bears a ratio of two small integers with the experimental value of the corresponding note. This approach can be applied to any musical instrument having same inner plate like structure.

1. Introduction

Pythagoras showed that under same tension, when two strings of same material with different length are made to vibrate the sound emitted would be pleasant to the ear provided the ratio of length is in a fixed ratio of two small integers [1,2]. Thus, a musical scale is a sequence of frequencies which have a particularly pleasing effect on human ear [3]. A widely used musical scale called a diatonic scale has eight frequencies covering an octave. Each frequency is called a note. The table below shows the frequencies in an octave [4].

Table 1: The Octave

Symbol	C	D	E	F	G	A	B	C_1
Indian Name	ସା (*Sa*)	ରେ (*Re*)	ଗା (*Ga*)	ମା(*Ma*)	ପା (*Pa*)	ଧା (*Dha*)	ନି (*Ni*)	ସା (*Sa*)
Frequency in Hz	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512

These notes are sound waves of different frequencies. Musicians, world over, speak of a musical tone in terms of three characteristics: loudness, pitch and quality [5]. Being a longitudinal wave sound wave travels in a medium as compressions and rarefactions. And the inverse of the time required for either a compression or a rarefaction in a particular direction to repeat itself is the pitch.

When the screw of a mouth organ is opened, small foil like plates of copper are seen, one end of which is tightly fixed and the other free. Each plate is connected to a hole through which air can be blown to set the plate into vibrations thereby producing sound [6-8]. Similarly, a cantilever is a beam made up of material like wood, iron etc whose one end is also fixed and the other left free to oscillate. For comparison purpose no mass is attached to the free end of the cantilever in this study.

2. Methodology

The time period of a cantilever is given by the formula

$$T = 2\pi\sqrt{\frac{\left(M + \frac{33}{140}ml\right)l^3}{Yab^3}}$$

where $M =$ mass attached at the free end of the cantilever, $m =$ mass of unit length of the cantilever material, $l =$ length of the cantilever, $Y =$ Young's modulus of the cantilever beam, $a =$ breadth of the cantilever, $b =$ thickness of the cantilever

Here, in this study $M = 0$, so the above formula reduces to $T = 2\pi\sqrt{\frac{33ml^4}{140Yab^3}}$ and the frequency becomes

$$f = \frac{1}{2\pi l^2}\sqrt{\frac{140Yab^3}{33m}} \quad (1)$$

Similarly, the screw of the mouth organ is loosened and measurement of length, breadth, and thickness of each plate corresponding to a particular note are taken as tabulated below (we have taken three sample notes of *Sa*, *Ga* and *Pa*)

Table 2: Length, breadth, and thickness of the three notes' (*Sa*, *Ga* and *Pa*) producing foils

Notes	Length in cm	Breadth in cm	Thickness in cm
ସା (*Sa*)	1.48	0.2	0.01
ଗା (*Ga*)	1.34	0.2	0.01
ପା (*Pa*)	1.20	0.2	0.01

3. Results and Calculations

All the eight plates of the mouth organ have the same values of a, b, m, and Y. So, using Eqn (1), we can calculate f_1 and f_2 respectively for *Sa* (ସା) and *Ga* (ଗା) as $f_1 = \frac{K}{l_1^2}$ and $f_2 = \frac{K}{l_2^2}$. So,

$$\frac{f_1}{f_2} = \left(\frac{l_2}{l_1}\right)^2 \qquad (2)$$

This is the relation which connects the mouth organ with the cantilever. By putting the values from table 2, we get $\frac{f_2}{f_1} = \left(\frac{l_1}{l_2}\right)^2 = \frac{219}{180} = 1.21$. Comparing it with the experimental values for the frequency of notes *Sa* and *Ga*, i.e., $\frac{f_2}{f_1} = \frac{320}{256} = 1.25$, the experimental error becomes 3%. Similarly, $\frac{f_3}{f_1} = \frac{l_1^2}{l_3^2} = \frac{219}{1.44} = 1.52$ । The actual ratio is $\frac{f_3}{f_1} = \frac{384}{256} = 1.50$ । And the percentage error is $\frac{0.02}{1.44} \times 100 = 1$ %. Comparing Pa and Ga, we get from cantilever as $\frac{f_3}{f_2} = \frac{l_2^2}{l_3^2} = \frac{1.80}{1.44} = 1.25$. The actual ratio is $\frac{f_3}{f_2} = \frac{384}{320} = 1.20$. Thus the percentage error is 3 %.

Now we will go another step to calculate the frequencies of different notes of mouth organs on the basis of cantilever. Each foil or plate is made up of copper. If ρ is the density of copper and

$m = \frac{M}{l} = \frac{\rho abl}{l} = \rho ab$. Putting this value in Eq (1) in the frequency formula we get $f = \frac{1}{2\pi l^2}\sqrt{\frac{140Yb^2}{33\rho}} = \frac{b}{2\pi l^2}\sqrt{\frac{140Y}{33\rho}}$.

For copper plates, Y =120× 10^9Nm^{-2} and ρ =8890 kg/m^3. So, $f = \frac{b}{6.28l^2}\sqrt{\frac{140\times120\times10^9}{8890\times33}} = \frac{b}{l^2}\times\frac{1}{6.28}\sqrt{\frac{5100\times10^7}{889}} = \frac{1206b}{l^2}$

From the formula it is seen that (i) The frequency of sound waves produced by the plates is independent of the breadth of the plates (a) (ii) it is directly proportional to the thickness of the plates (b) (iii) it is inversely proportional to the square of the length of the plates (l^2). Let us take the note (ଗା) *Ga*. For (ଗା)*Ga*, $b =$ 0.012cm, l=1.34cm and f becomes $804\ Hz$. But from the data of (ଗା) *Ga* (Ref table 1) it is 320 Hz. And the ratio is $\frac{804}{320} = \frac{5}{2}$. For (ସା) *Sa*, $b =$ 0.012cm, l =1.4 8cm and $f = 1206 \times \left(\frac{0.012\times10^{-2}}{(1.48\times10^{-4})^2}\right) = 660.7\ Hz$. So, the ratio is $\frac{661}{256} \cong \frac{5}{2}$. This verifies the Pythagoras' observation [1], i.e., when two strings of same material with different length are made to vibrate the sound emitted would be pleasant to the ear provided the ratio of length is in a fixed ratio of two small integers.

4. Conclusion

The ratio of frequencies of two notes calculated assuming foils in the mouth organ to be cantilevers, we arrived at a remarkable agreement with the ratio of experimental values of the corresponding notes. Again, the ratio of the value of the frequency of a note with cantilever and its experimental value bears a ratio of two simple integers.

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